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by

STEPHEN C. GRAVES

Technical Report No. 169

OPERATIONS RESEARCH CENTER

MASSACHUSETTS INSTITUTE
OF
TECHNOLOGY

November 1979

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Operations Research Center
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FOREWORD

The Operations Research Center at the Massachusetts Institute of Technology is an interdepartmental activity devoted to graduate education and research in the field of operations research. The work of the Center is supported, in part, by government contracts and grants. The work reported herein was supported by the Office of Naval Research under Contract N00014-75-C-0556.

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ABSTRACT

Production scheduling may be defined as the allocation of available production resources over time to best satisfy some set of criteria. Typically, the scheduling problem involves a set of tasks to be performed, and the criteria involve tradeoffs between early and late completion of a task, and/or between holding inventory for the task and frequent production changeovers. The intent of this paper is to present a broad classification for various scheduling problems, to review important theoretical developments for these problem classes, and to contrast the currently available theory with the practice of production scheduling. This paper will highlight problem areas both for which there is a significant discrepancy between practice and theory, and for which the practice corresponds closely to the theory.

1. Introduction

Production scheduling may be defined as the allocation of available production resources over time to best satisfy some set of criteria. Typically, the scheduling problem involves a set of tasks or requirements to be performed, where the criteria might involve tradeoffs between early and late completion of a task, and/or between building inventory for an item and frequent production changeovers. The intent of this paper is to present a broad classification for various scheduling problems, to review important theoretical developments for these problem areas, and to contrast the currently available theory with the current practice of production scheduling.

One way to introduce and summarize the contents and scope of this paper is to indicate what the paper has not tried to accomplish. First, we have no intentions of giving an exhaustive survey of the literature in production scheduling. To do so would not only be presumptuous, but also potentially repetitive of previous survey and review papers such as Moore and Wilson [99], Elmaghraby [37], Bakshi and Arora [10], Day and Hottenstein [29], Panwalker and Iskander [106], Salvador [114], Godin [55], and Eilon [34]. Extensive bibliographies are also available from the books by Muth and Thompson [101], Conway, Maxwell, and Miller [25], Eilon and King [35], Elmaghraby [38], Baker [5], Lenstra [82], Rinnooy Kan [110], and Coffman [23]. Rather, the paper attempts only to highlight the status of current research on classical production scheduling problems. The emphasis of the paper is more on establishing a current perspective on the status of scheduling research relative to scheduling practice.

Second, the paper largely ignores the recent work in complexity theory for scheduling algorithms and problems. We felt that a proper treatment of this material would detract from the primary focus of the paper, that being the status of current theory versus practice. Again, excellent bibliographies for this material can be found in Coffman [23], and Garey, Graham, and Johnson [47].

Third we have limited this paper to a fairly restrictive definition of production scheduling so as to minimize the overlap with the review papers on inventory management by Silver [122] and on production planning by Hax and Meal [64]. We distinguish production scheduling from inventory management based on assumptions for the production resources; the allocation decisions for the production resources are the primary focus of the production scheduling problem, whereas these decisions are made exogenously in the inventory management context. We distinguish production scheduling from production planning based again on the assumptions for the production resources; the determination of the production resource level is exogenous to the production scheduling problem, but is a primary decision in the production planning process.

The remainder of the paper is organized into four sections. In the next section a classification scheme is given. Section 3 reviews the status of production scheduling theory according to the classification previously given. Section 4 presents some observations on the practice of production scheduling, while the last section suggests some directions for future research based on these observations.

2. Problem Classification

Numerous schemes have been proposed for categorizing production scheduling problems. The intent of any classification is to provide a semblance of organization so that the major differentiating dimensions of the problem classes are identified. For our purposes we desire a broad classification which allows us to encompass the general characteristics of both scheduling theory and scheduling practice. With this in mind, we propose the following three dimensions for classifying production scheduling problems:

1) requirements generation
2) processing complexity
3) scheduling criteria.

These dimensions, as will be seen, are imperfect; the dimensions are not independent, they do not guarantee a unique representation for any problem setting, and they may be ambiguous at times. However we hope to show that they are adequate for structuring our discussions and for aiding comparisons across problems.

The first dimension, requirements generation, is a key distinction. Requirements may be generated either by open orders or by inventory replenishment decisions. This distinction is often made in terms of an <u>open shop</u> versus a <u>closed shop</u>. In an open shop all production orders are by customer request, and no inventory is stocked; in a closed shop all customer requests are serviced from inventory, and the production tasks are in general a result of the inventory replenishment decisions. The production scheduling problem is quite different depending on the requirements generation. For the open shop, production scheduling in its simplest form is a sequencing problem in which the open orders are to be sequenced at each facility. For the closed shop, production scheduling must be involved not only in the sequencing decisions, but also in the lot-sizing decisions associated with the inventory replenishment process. Although a pure open or a pure closed shop is rare, most production environments are primarily either open or closed. For this review we focus on this distinction; furthermore we assume that the decision as to whether a shop is open or closed has been made.

The second dimension, processing complexity, is concerned both with the number of processing steps associated with each production task or item, and with the existence of alternative routings for a particular task. A common breakdown for this dimension is as follows:

- one stage, one processor (facility)
- one stage, parallel processors (facilities)
- multistage, flow shop
- multistage, job shop.

The one stage, one processor problem, which is also termed the one machine problem, is the simplest problem form; here all tasks require one processing step which must be done on the one production facility. The one stage, parallel processor problem is similar to the one machine problem except that now each task requires a single processing step which may be performed on any of the parallel processors. For the multistage problem, each task requires processing at a set of distinct facilities, where typically there is a strict precedence ordering of the processing steps for a particular task. The flow shop problem assumes that all tasks are to be processed on the same set of facilities with an identical precedence ordering of the processing steps. The job shop problem is the most general production scheduling problem in the classification; here there are no restrictions on the processing steps for a task, and alternative routings for a task may be allowed. The above breakdown for processing complexity, although by no means exhaustive of all possibilities, seems to be fairly representative for most production environments. This may be due to the vast generality of the multistage, job shop category; nevertheless, the simpler categories are of importance in that many shops which are limited by a bottleneck department or a bottleneck line may be viewed as a one stage problem or as a flow shop. Furthermore, theoretical insight from these simpler problems is often the first step in tackling the more complex problems.

The third dimension, scheduling criteria, indicates the measures upon which schedules are to be evaluated. Two broad classes of criteria are schedule cost and schedule performance. The cost associated with a particular schedule includes the fixed costs associated with production setups or changeovers, variable production and overtime costs, inventory holding costs, shortage costs for not meeting deadlines or for stocking out, and possibly expediting costs for implementing the schedule in a dynamic environment. The system costs for generating the schedule and for monitoring the progress of the schedule also need to be included in the schedule costs. The performance of the schedule may be measured in many ways. Common measures are the utilization level of the production resources, the percentage of late tasks, the average or maximum tardiness for a set of tasks, and the average or maximum flow time for a set of tasks.[1] In addition, for a closed shop, service criteria, such as the percentage of demand filled from stock, may be used for evaluating a production schedule. In most production environments, the schedule evaluation is based on a mixture of both cost and performance criteria; however, as will be seen, most of

[1] Tardiness is the positive part of the difference between a task's actual completion time and its desired completion time. The flow time for a task is the difference between the completion time of the task and the time at which the task was released to the production shop.

the theoretical literature on production scheduling addresses single criterion problems. Indeed, the literature for open shops deals primarily with schedule performance criteria, whereas the closed shop literature is concerned with a minimum cost criterion.

Two other dimensions which might have been included here are the nature of the requirement specification and the scheduling environment. Depending upon how the requirements are generated, the specification of the requirements may be termed deterministic or stochastic. For instance, for an open shop, the processing time for each step of the task may be known, or may be a random variable with specified probability distribution. Similarly, for a closed shop, the customer demand process, which drives the inventory replenishment decisions, may be assumed to be stochastic or deterministic. The scheduling environment deals with the assumptions on the availability of information on future requirements. A common distinction is made between a static and dynamic environment. In a static environment, the scheduling problem is defined with respect to a finite set of fully-specified requirements; no additional requirements will be added to this set, nor will any of the specifications be altered. As a contrast in a dynamic environment, the scheduling problem is defined not only for the known requirements, but also with respect to the anticipations for additional requirements and specifications generated over the future time periods. Both the nature of the requirements specification and the scheduling horizon are primarily model considerations, as opposed to problem characteristics. For most production environments the scheduling problem is stochastic and dynamic; most models for scheduling problems, however, are inherently deterministic and static. Most scheduling systems are implemented as if the production environment were deterministic and static over a specified finite horizon. Our review of production scheduling theory and practice will focus almost entirely on results from deterministic and static models for production scheduling problems. Hence we do not formally include these distinctions in this problem classification, but do acknowledge the importance of these distinctions for characterizing models.

3. Review of Production Scheduling Theory

This section gives an overview of major work in production scheduling theory. The organization of this section follows the classification scheme presented in the previous section. We reemphasize that the primary focus of the review is on reported work for the deterministic, static problems.

3.1 Open Shop Problem

The open shop scheduling problem, also called the job shop scheduling problem, may be broadly defined as having to sequence a family of processors so as to complete a given set of tasks and optimize some performance measure. These problems are all combinatorial problems of varying difficulty, and they may all be solved, in theory, by an enumeration strategy such as a branch and bound procedure. As we will see, however, these problems range quickly from the trivial to the impossible.

One Stage, One Processor

The one stage, one processor problem, or one machine problem, has been the most popular of all scheduling problems. A wide variety of results exist for a vast set of problem specifications. The best known of these results are the procedures for minimizing the mean flow time (Smith [127]) and for minimizing the maximum tardiness (Jackson [71]); both of these procedures determine the optimal task sequence by means of a simple ordering of the tasks. A slightly more complex procedure is that of Moore [98] for minimizing the number of late tasks; here the tasks are first ordered according to their desired completion time, and then this sequence is modified by the sequential removal of late tasks.

While the above-mentioned problems have proved to be quite easy, the problem of minimizing weighted tardiness has been considerably more difficult. Indeed this problem has been shown to be NP-complete by Karp [75]. Lawler [81], Emmons [40], Schwimer [121], Fisher [43], Srinivasan [128], Rinnooy Kan, et al [111], Picard and Queyranne [108], and Baker and Schrage [9], to mention a few, have all studied the one-machine tardiness problem or a generalization of it. The recent work of Schrage and Baker [116] seems to have substantially tamed this problem. They formulate the problem as a general dynamic program; they then perform the recursive computations very efficiently by exploiting the dominance properties introduced by Emmons [40], and by an ingenious implementation of the algorithm. They report solving both 50-task tardiness problems and 20-task weighted tardiness problems in less than one second of cpu time on an IBM 370/168.

One Stage, Parallel Processors

The one stage, parallel processor problem is an important generalization of the one-machine problem; whereas the one-machine problem is primarily of theoretical interest, the one stage, parallel processor scheduling problem occurs in many settings such as continuous processing plants for the glass and chemical industries, and computer installations. Unfortunately, very few of the simple results and algorithms for the one-machine case can be carried over to the parallel processor problem. Most of the results obtained assume that the processors are identical. A distinction is also made for problems with preemptive versus nonpreemptive tasks.[1] The most common criteria studied for the problem are weighted flowtime, maximum flowtime (or makespan), and weighted tardiness.

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[1] A task is said to be preemptive if the processing of the task may be split either on a single processor or across several processors.

For the weighted flowtime criterion, McNaughton [95] shows that an optimal schedule exists in which no tasks are preempted or split; hence the distinction is not important for this criterion. To minimize mean flowtime on identical processors, a simple sorting procedure, analogous to the shortest processing time procedure for the one-machine case, is optimal [95]. When the processors are not identical, Horn [68] shows how to solve the mean flowtime problem as a transportation problem. Unlike the one-machine problem, here the weighted flowtime problem is considerably harder than the mean flowtime problem. Eastman, et al [33] give a lower bound for the weighted flowtime criterion, Rothkopf [113] formulates the problem as a very general dynamic program, and Baker and Merten [7] propose and test several heuristics for this problem.

McNaughton [95] gives a very simple solution for minimizing the maximum flowtime when the processors are identical and preemption is allowed. Special cases for the maximum flowtime criterion have also been solved assuming that the processors are identical and all tasks have the same processing times. Hu [69] gives a list scheduling procedure for the nonpreemptive case where the precedence relationships for the tasks form an assembly tree structure. Coffman and Graham [24] present a similar solution procedure for the nonpreemptive case with general precedence relations and two identical processors. Both of these procedures can be extended to allow general processing times for preemptive tasks.

No simple results seem to exist for minimizing weighted tardiness. Optimal search algorithms for special cases have been given by Root [112], Elmaghraby and Park [39], Barnes and Brennan [12], and Nunnikhoven and Emmons [104]. Lawler [81] considers the identical processor problem where all tasks have the same processing time, and shows how to formulate this problem as a transportation problem; indeed, for this special case any criterion that is dependent on completion time may be modeled as such.

An interesting analogy exists between the one stage, parallel processor problem and the vehicle routing problem (e.g. Clarke and Wright [22]). With a criterion of minimizing changeover cost (or time) where a task's changeover cost may be sequence dependent, the scheduling problem with m identical processors is equivalent to a vehicle routing problem with m identical vehicles in which total travel distance is to be minimized. Here, the tasks correspond to the cities to be visited in the vehicle routing problem. Parker, et al [107] have used this equivalence for finding heuristics for this parallel processor problem.

Flow Shop

The flow shop problem is the simplest multistage scheduling problem; unfortunately, as we will see, it has proven to be quite formidable. The problem is to determine how to sequence the set of tasks on each processor, where the processors are arranged in series and each task must visit each processor in the prescribed order. A common assumption made by most researchers is to restrict attention to schedules for which the sequence of tasks is identical on each processor. Such schedules are called permutation schedules, and have been shown to be optimal for all 2-processor problems, and for the 3-processor problem with a maximum flowtime criterion; however, in general, permutation schedules need not be optimal. Most of the reported work has also been limited to considering the maximum flowtime criterion, with nonpreemptive schedules.

The best known result for flow shop scheduling is that of Johnson [74] for the 2-processor problem. He presents a simple list scheduling procedure for this problem for minimizing the maximum flow time. The simplicity of this procedure has enticed others to spend countless hours seeking an analogous procedure for the 3-processor problem, without success. Indeed, this problem has been shown to be NP-complete for nonpreemptive schedules by Garey, et al [48] and for preemptive schedules by Gonzalez and Sahni [56].

Numerous combinatorial optimization procedures have been proposed for solving the general flow shop problem with the maximum flow time criterion (e.g. Ashour [1], Gupta [63], Ignall and Schrage [70], Lomnicki [84], McMahon and Burton [93], Smith and Dudek [126], and Szwarc [132]). The most successful approaches seem to be branch and bound procedures which use both bounds and "elimination" methods for eliminating dominated sequences. Both Baker [3] and Lageweg, et al [77] have performed comparative computational studies of the relative efficacy of the various bounding and elimination strategies. The most recent of these studies [77] finds that a bound based on Johnson's 2-processor procedure, combined with an elimination criterion of Szwarc [132] gave the best performance over a wide range of test problems. Using this procedure, Lageweg, et al [77] were able to solve more than half of their test problems, each with twenty tasks to be scheduled on three processors, in less than 0.4 seconds of cpu time per problem on a Control Data Cyber 73-28. However, when the number of processors was increased to five, their procedure was unable to solve half of the test problems within the one minute of cpu time allowed per problem.

Paralleling the work on optimal procedures for the flow shop problem has been work on heuristic procedures. Noteworthy heuristics for the maximum flow time criterion are those of Campbell, et al [19] and Dannenbring [28]. The heuristic of Campbell, et al [19] uses Johnson's procedure to solve a series of 2-processor approximations to the actual problem with m processors. The heuristic then chooses the best schedule from these approximations. Dannenbring [28] also uses Johnson's procedure to solve a

2-processor approximation to the actual problem; the generated task sequence is then improved as much as possible by considering switching adjacent tasks in the sequence. In an extensive evaluation of a wide set of heuristics, Dannenbring [28] found that his heuristic performed the best on average; on a sample set of small problems, this heuristic obtained the optimal schedule for over 75% of the problems, and had an average deviation from the optimum schedule of less than 1%. The heuristic procedure of Campbell, et al [19] obtained only 55% of the optimal schedules with an average deviation of 1.7% from optimum for the same set of problems; however, their heuristic was ten times faster than that of Dannenbring.

Job Shop

The job shop scheduling problem is the most general and the most difficult of the open shop problems. Here there are no restrictions on the requirements associated with each task, so that a task may require processing on any subset of processors in any conceivable order. Most researchers have assumed that all tasks are nonpreemptive and that the criterion is to minimize the maximum flow time. All optimization approaches to this problem seem to be branch and bound procedures (e.g. Ashour, et al [2], Balas [11], Brooks and White [17], Charlton and Death [21], Fisher [44], Florian, et al [46], Giffler and Thompson [53], Greenberg [62], Lageweg, et al [78], and Schrage [115]) where the various procedures differ primarily with respect to the branching rules, the bounding mechanism, and the generation of upper bounds. Despite the preponderance of effort on this problem, the largest problems reported to have been solved have less than ten tasks to be scheduled on less than ten processors. Indeed, while Lageweg, et al [78] report solving a difficult six task, six processor problem within a few seconds, they (and presumably anyone else) are unable to solve a ten task, ten processor problem within five minutes of cpu time on a Control Data Cyber 73-28.

There have been two distinct heuristic approaches to the deterministic, static job shop scheduling problem. The first is a construction approach (e.g. Jeremiah, et al [73], Gere [51]) in which a feasible schedule is constructed by scheduling all tasks as early as possible; conflicts arising from more than one task being ready for processing at a particular station, are resolved by a specified dispatching rule such as selecting that operation with the shortest processing time or with the most operations remaining. The second approach is random sampling (e.g. Giffler, et al [54]), where a sample of feasible schedules are generated with the best schedule being chosen. The schedules are generated by construction with conflicts being resolved randomly. A similar approach has been taken by Heller [67] for the flow shop problem.

A wide body of research exists for the dynamic job shop problem. The primary effort has consisted of simulation studies of particular job shop settings in which a variety of local dispatching rules are compared on a set of criteria. It is beyond the scope of this paper to review this work. The reader is referred to the book by Conway, Maxwell, and Miller [25], and to the review papers of Moore and Wilson [99], Day and Hottenstein [29], and Panwalker and Iskander [106].

3.2 Closed Shop Problem

The closed shop scheduling problem is to find a production schedule to satisfy some given requirements at minimum production cost, where the production schedule specifies both the run quantities for a set of items and the setup sequences for a set of facilities. This problem is more commonly known as the lot-sizing problem. The requirements to be met are characterized by specifying for a set of items the assumed demand process, which we take to be deterministic. The production-related costs, for models with deterministic requirements, usually consist of inventory holding costs, fixed setup costs, and variable production costs. A common distinction is made between models that assume demand to be constant over time, and models which allow time-varying deterministic demand. In the latter case, production decisions are made for discrete points in time, whereas the former allows decisions to be implemented at any point in the time continuum. All of the deteministic closed shop scheduling problems can be formulated as mixed integer linear programs; most of the solution procedures that we will see, are enumeration procedures which attempt to exploit some special structure of these programs.

<u>One Stage, One Facility</u>

The lot-sizing problem with one facility has been examined by many researchers under a variety of assumptions. The multi-item problem for constant demand with item costs consisting of a setup cost and a linear inventory holding cost, is known as the economic lot scheduling problem; Elmaghraby [36] provides an excellent review of this problem. A common assumption is to restrict attention to base period policies of the form $(k_1,k_2,\ldots,k_n;\ T)$ where k_i is integer for $i=1,\ldots,n$ and n is the number of items to be scheduled; the policy is a cyclic policy with a base period of length T where item i is produced once every k_i periods. A particular base period policy $(k_1,\ldots,k_n;\ T)$ is feasible if a cyclic production schedule can be found such that the production requirements in a given period do not exceed the length of the period T. The most common solution procedure (e.g. Doll and Whybark [30], Elmaghraby [36], Madigan [88], Stankard and Gupta [129]) is first to find a good candidate policy ignoring production capacity; this candidate policy is then tested for feasibility, and if infeasible, modified to obtain a feasible policy.

The joint replenishment problem is another version of the one facility problem that is closely related to the economic lot scheduling problem. Here demand is again assumed to be constant, but now there are economies of scale from the joint replenishment of several items; these economies of scale are characterized by assuming a major setup cost associated with initiating production on the facility, and minor setup costs for switching from one item to another. Most researchers (e..g Goyal [57],[58], Nocturne [103], Shu [119], and Silver [123]) have focused on finding good base period policies $(k_1,\ldots,k_n; T)$ where now $1/T$ is the frequency of major setups. For both the economic lot scheduling problem and the joint replenishment problem, Graves [60] has shown that the determination of the base period policy, ignoring production feasibility, is equivalent to the lot-sizing problem for a two-echelon distribution system studied by Graves and Schwarz [61]. Hence any solution procedure for one problem may be immediately adapted to the other two problem classes.

The analogous version of the one facility production scheduling problem in which demand is deterministic, but time-varying, is the capacitated version of the well-known lot-sizing problem studied by Wagner and Whitin [135]. For a single-item problem with a linear inventory holding cost, convex production cost and convex shortage cost, Eppen and Gould [41] use a Lagrangean relaxation to develop a forward algorithm for finding the optimal schedule. Florian and Klein [45] also consider a single-item problem, but with constant capacity, and a concave production cost. They give a dynamic programming formulation which requires the solution of $T(T-1)/2$ shortest path subproblems for T being the number of periods in the scheduling horizon. This formulation has been extended and/or generalized by Jagannathan and Rao [72], Swoveland [131], and Lambrecht and Vander Eecken [79]. Baker, et al [6] present a tree-search solution procedure for a similar problem to that of Florian and Klein, but with time-varying production capacity; they report extensive computational experience on an IBM 370/158 in which a twelve-period problem was solved on average in less than 0.25 seconds cpu time, while a 24 period problem took up to 4.50 seconds cpu time to solve. Heuristics for single-facility constrained lot-sizing problems have been proposed for the single-item case by Silver and Dixon [124] and for the multi-item case by Manne [89], Eisenhut [35a], Newson [102], and Van Nunen and Wessels [133]. On a sample of over 600 test problems, Silver and Dixon [124] report that their heuristic gave the optimal solution in 83% of the cases, and when it did not find the optimum, the average cost penalty was only 1.7%.

One Stage, Parallel Facilities

The one stage lot-sizing problem on parallel facilities has received less attention than its single facility analog. However the reported research on this problem seems to be more directly connected with actual scheduling problems than that for the single facility problem. Dorsey, et al [31],[32], and Ratliff [109] examine in a series of papers the scheduling problem in which several items are to be produced on a set of identical parallel facilities over a finite horizon; demand is time-varying and given by period. The schedule costs may include a convex production cost, and linear inventory holding and backorder costs. Production is by batches, where an item's batch size is set equal to one period of production. With this restriction the problem may be formulated as a minimum cost network flow problem for which efficient solution procedures exist.

Love and Vemuganti [85] obtain a similar result in a slightly different context. They also consider the problem of scheduling items on identical parallel facilities, but with production requirements being specified solely in terms of the minimum number of batches required by item for each period. The objective is to minimize total changeover costs where a changeover occurs whenever a facility is switched from one item to another. Love and Vemuganti formulate the problem as an integer linear program, and show how it may be transformed into a minimum cost network flow problem.

Both Klingman, et al [76] and Caie, et al [18] consider a single stage, parallel facility lot-sizing problem in which item demand is now assumed to be constant over a finite horizon. They both assume each item is produced entirely on one machine at regular intervals. For instance, Caie, et al [18] have a sixteen-week scheduling horizon in which the time between production runs for any item is restricted to one week, two weeks, four weeks, or eight weeks. The scheduling objective is to minimize total costs, consisting of setup costs, inventory holding costs, and storage costs. The problem is modeled as an integer linear program with decision variables denoting for each item both the machine to which it is assigned, and the item's production frequency. The program solution does not sequence the items on the parallel machines, but rather provides schedule for machine loading. The integer linear program is solved by a branch and bound procedure, in which bounds are generated from a subgradient optimization of a Lagrangean relaxation of the program.

Multistage Systems

The multistage lot-sizing problem is often characterized by the network describing the processing steps for each item. The counterpart of the flow shop for the open shop problem is the serial system for the closed shop problem. Here, the item requires several processing stages where the stages are strictly ordered. The lot-sizing problem is to find a feasible production schedule that meets the demand requirements at minimum total cost where there may be an inventory holding and production cost at each stage.

For the serial-system lot-sizing problem with uncapacitated stages, concave production and holding costs, and time-varying demand, Zangwill [137] shows that the optimal solution is contained in the set of extreme flows of a single-source network, and gives a dynamic programming procedure for calculating the optimal policies. Love [86] also considers the serial-system problem and presents an alternative dynamic programming algorithm which exploits the nested property of the solution. The amount of computation for both procedures is bounded by a polynomial in the number of stages and the number of time periods. Lambrecht and Vander Eecken [80] examine a single-item serial system with time-varying demand for which the last processing stage is capacitated. They characterize the optimal solution in terms of extreme network flows and propose a decomposition solution procedure in which the problem is separated between the capacitated stage and the earlier uncapacitated stages. The solution procedure is to enumerate all extreme point schedules for the capacitated stage, and then use Zangwill's algorithm for scheduling the uncapacitated stages.

The analog to the general job shop problem would seem to be the lot-sizing problem for an assembly network. Here the production of an item is characterized by a directed, connected graph where each node has at most one successor; each node corresponds to a component or subassembly of the item, while the arcs denote how the item is assembled. The serial system is a special case.

Veinott [134] and Crowston and Wagner [26] formulate the lot-sizing problem for an uncapacitated assembly network with time-varying demand, concave production costs and linear inventory holding costs, as a dynamic program by exploiting an extreme point characterization of the optimal solution. However, these algorithms are quite complex, with the amount of computation increasing exponentially with the number of periods in the scheduling horizon. A variety of single-pass heuristics (e.g. McLaren [91], McLaren and Whybark [92], Biggs, et al [14], Blackburn and Millen [15]) have been proposed for this problem. These heuristics decompose the assembly network into echelons so that a production schedule may be generated by solving sequentially a series of single-stage lot-sizing problems. Graves [59] presents and tests a multipass heuristic in which a schedule, constructed by a single-pass procedure, is revised iteratively based on local shadow-price information. On a series of 250 test problems, each with five stages, the multipass heuristic obtained the optimal solution in over 90% of the cases, with the average percentage cost deviation being less than 0.5% from optimum.

For the lot-sizing problem for assembly systems with constant demand over an infinite horizon, Crowston, et al [27] show that an optimal scheduling policy has stationary lot sizes such that each stage's lot size is an integer multiple of the lot size for the unique successor of that stage. This characterization of the optimal policy form is then used by Schwarz and Schrage [118] to formulate the lot-sizing problem as an integer program which is solved by branch and bound. Schwarz and Schrage [118] also suggest a "system myopic" heuristic procedure based on the optimal solution procedure of Schwarz [117] for the lot-sizing problem for a two-stage serial system; on a series of 3000 small test problems the heuristic gave the optimal policy in over half the cases, and had an average cost error of less than 5%.

4. Observations on Practice

The previous section has reviewed the current theoretical work on the classical production scheduling problems. As we have seen, there have been many significant advances in production scheduling theory, yet there are still many unsolved problems with opportunities for improvements. Unfortunately, it is not so easy to review and summarize production scheduling practice. Actual scheduling environments are somewhat illusive to define rigidly and consequently very difficult to classify; indeed, most production schedulers will claim, with some justification, that their scheduling setting is not only unique, but sufficiently different from any other setting to require a problem-specific solution. In addition to the problem diversity, practitioners have less incentive than theoreticians to report upon their work in production scheduling. This is due to the perceived limited generality and to the proprietary nature of their work. Hence a review of production scheduling practice is not attempted here; rather, we offer observations as to the state of the practice of production scheduling. The intent of these observations is not to give a definitive characterization of scheduling practice, but rather to establish a perspective with which to contrast scheduling theory.

The first observation is the predominance of purely manual scheduling systems, especially in relatively simple production environments involving only a few processing steps. These systems rely primarily on the expertise of a few experienced schedulers who construct, revise, and maintain the production schedule using no more than a few graphical aids such as a Gantt chart. To an observer, at least, it is often not clear how exactly a schedule is constructed nor how alternative schedules are compared or evaluated. The schedule evaluation seems to be qualitative and to depend upon several criteria. The dominant schedule criterion is often schedule feasibility, although many other criteria such as schedule flexibility may be important. Nevertheless, such systems seem to work in that the generated schedules are viewed as being quite satisfactory. This, however, is difficult to ascertain due to the qualitative nature of the criteria.

Many complex multistage production environments have implemented for their production scheduling either shop floor control systems or material requirements planning (MRP) systems or both (e.g. Orlicky [105]). These systems are essentially logical systems, which given today's computer power, are able to perform both detailed bookkeeping and extrapolation functions. These systems do not inherently make scheduling decisions; they have no mechanisms for considering the standard tradeoffs associated with scheduling decisions. Rather, these systems provide better and more current information to the production controllers so that they may make the appropriate task-sequencing and item lot-sizing decisions. It is common for the production controllers to defer these decision-making responsibilities by selecting a scheduling algorithm to be built into the information system. In the case of the shop floor control systems, this corresponds to selecting a local dispatch rule for sequencing tasks waiting at each production processor; common dispatch rules are to sequence the tasks according to expected processing times, or slack times[1] or the ratios of the tasks' remaining processing time to the slack time. For a material requirements planning system, a lot-sizing procedure must be chosen.

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[1] A task's slack time is the difference between the time remaining until the desired completion time and the remaining processing time.

Typically, the system will employ a single-stage, uncapacitated lot-sizing procedure which is used to schedule sequentially the production facilities. Examples of common lot-sizing procedures are given by Berry [13] and Silver and Meal [125]. It should be noted that these procedures ignore production capacities, and hence cannot guarantee production feasibility. Consequently, the schedule generated from these systems may need to be manually adjusted.

Both material requirements planning systems and shop floor control systems have been widely implemented, and have been credited with producing significant cost savings and performance improvements. Indeed, these systems seem to have revolutionized how a production shop is run. It is not clear, however, as to how much of the improvements are attributable to having better information compared with using various sequencing or lot-sizing procedures.

The above observations suggest that the current operations research techniques for scheduling may be either mismatched, inadequate, or not needed for many production settings. This is only partially true. We are beginning to see more scheduling implementations which rely upon operations research tools to examine tradeoffs and assure schedule feasibility. In particular, this seems to be true in single-stage, parallel-facility production shops. Here, the scheduling problem may be formulated as a mathematical programming problem, which often is tractable as a result of its size and structure. Examples of such applications have been reported by Geoffrion and Graves [52], Love and Vemuganti [85], Caie, et al [18], and Ferreira and Hodgson [42].

A slightly different observation applies to settings in which a formal aggregate production planning procedure exists. Here the scheduling problem may be sufficiently constrained by the aggregate plan so that it reduces essentially to a disaggregation scheme in which some secondary criterion is optimized subject to satisfying the aggregate plan. Common disaggregation rules are to equalize the run-out time across items or to minimize the expected shortage cost for the immediate period. Illustrations of such applications are given by Holt, et al [67a], Hax and Meal [65], Meal [96], and Blake [16].

In summary, there is anything but a one-to-one correspondence between scheduling theory and practice. For complex multistage production settings, the theory is not sufficiently developed to be immediately applicable. Furthermore, some of the scheduling theory is mismatched to the needs of the production scheduler. Nevertheless, as noted, there are encouraging signs for certain production settings in which current operations research methodology has proved useful. Clearly, research opportunities exist for remedying some of the discrepancies between the theory and the practice, and for expanding upon the existing successes with applying scheduling theory. Some of these opportunities are discussed in the next section.

5. Future Directions

We have seen that there is a gap between production scheduling theory and practice. Future research efforts must be addressed so as to reduce these differences. Much of the previous research effort has been spent on developing more powerful algorithms and/or more effective heuristics for the standard production scheduling problems. These efforts clearly need to be continued; the development of better algorithms and heuristics is essential if scheduling theory is to continue to improve scheduling practice. However, there are other research directions requiring more attention, which may have as great an impact on scheduling practice. There is a great need, not only for better scheduling algorithms, but both for more realistic models of the scheduling setting and for increased understanding of the dynamics inherent in the scheduling environment. Six areas for future research are suggested below.

A. Diagnostics: There is a need to be able to diagnose and evaluate an operating production scheduling system to determine whether the system is effective and whether the system can be improved. The diagnostic would be the first step in the analysis and revamping of a scheduling system. The diagnostic should be simple, accurate, and suggestive for the next, more-detailed step in the analysis. No general diagnostics seem to exist for scheduling systems; however, some preliminary work illustrating the use of diagnostics for a production planning system has been done by Hax, et al [66].

B. Scheduling Robustness: A frequent comment heard in many scheduling shops is that there is no scheduling problem but rather a rescheduling problem. It may be quite easy to construct a schedule; what is difficult is the constant schedule revision required by the dynamic environment. Hence, it is important to understand how to implement a schedule when shop conditions are uncertain. In particular, a need exists for understanding schedule robustness. We desire scheduling methods which either explicitly reflect the uncertain nature of the available information or give some guarantee as to the insensitivity of the schedule to future information.

In addition to the wide recognition of the need for schedule robustness, there is some very encouraging work in this area. One line of research is to characterize planning horizons for deterministic scheduling models, where a planning horizon establishes the insensitivity of the current scheduling decisions to future information beyond the horizon. Wagner and Whitin [135] have given the best-known such results for the uncapacitated, single-stage lot-sizing problem; Zabel [136] and Lundin and Morton [87] provide important extensions to this work. Morton [100] has also found conditions for determining planning horizons for a class of single-stage, convex-cost scheduling problems, extending the earlier work of Modigliani and Hohn [97], and Lieber [83]. The need exists for similar planning horizon results for a wider class of production scheduling problems.

A related line of research on scheduling robustness is the examination of the effect of the length of the scheduling horizon. Most scheduling models attempt to optimize the schedule with respect to a specified finite horizon, despite the fact that the schedule is to be implemented in a setting which may operate indefinitely. Typically, only the model decisions for the immediate period are implemented; in the next period the schedule is reoptimized based on revised and additional information, and again only the immediate period's decisions are used. In this fashion the finite-horizon scheduling model is used to generate a "rolling" schedule. Some interesting work has been performed on studying rolling schedules for two relatively simple scheduling models. Baker [4] presents an experimental study in which the effectiveness of rolling schedules produced for the uncapacitated, single-stage lot-sizing model, is examined. He finds that over a wide range of conditions, rolling schedules performed very well with system costs usually being within 10% of costs for an optimal schedule. Baker and Peterson [8] develop an analytical model for evaluating rolling schedules generated by a single-stage production model with quadratic production and inventory costs. They also find that under reasonable conditions such schedules are quite robust.

A third topic of research is concerned with system "nervousness" created by the dynamic scheduling environment. System nervousness is a consequence of the rolling schedule procedure in that schedule plans for future periods are repeatedly being changed; this instability can be quite costly if the schedule is being used as a basis for manpower planning and procurement decisions. Some prelminary work attempting to incorporate these costs into the scheduling decision is reported by Carlson, et al [20]. They consider the uncapacitated, single-stage lot-sizing problem and propose a simple modification to reflect the cost of schedule changes.

C. Schedule Interaction: Production schedules are implemented not in isolation, but as part of a total operating system. For instance, scheduling decisions affect and are affected by capacity planning decisions, various marketing decisions such as product promotions, and transportation/distribution schedules. This interaction is not reflected in most scheduling models. Some exceptions to this exist, though, especially with respect to the interaction of capacity planning with scheduling. For one, Hax and Meal [65] propose a hierarchical framework for a batch-processing environment for linking the capacity planning decisions with the detailed scheduling decisions. Shwimer [120] and Gelders and Kleindorfer [49],[50] present procedures for coupling capacity planning decisions with sequencing decisions in a single-facility open shop. Finally, Maxwell and Muckstadt [90] present a model and algorithm for coordinating production decisions with transportation decisions, where the production decisions involve both capacity planning and scheduling decisions. The above models are important in their recognition of the interaction between the scheduling decision and other decisions; still more effort is needed, though, to incorporate fully this interaction with the present scheduling models.

D. Value of Information: Given the predominance of material requirements planning systems and shop floor control systems, the question arises as to how valuable is the information from such systems for generating good schedules. Since both systems are essentially very elaborate information systems, we desire to understand the tradeoff between increased system costs for obtaining and maintaining accurate information versus the cost savings from improved schedules from these systems; indeed, research is needed for determining how elaborate such systems need be, and at what point is there a net negative return from more detailed information. There seems to be very little work on these questions.

E. Specialized Scheduling Functions: Whereas the primary focus of this review of production scheduling has been on sequencing and lot-sizing decisions, the scheduling function certainly encompasses other decisions which may be worthy of study. In particular, important questions exist with regard to the expediting of "hot" tasks and the releasing of new tasks to the shop floor. Expediting is a common practice in many control systems; however, its consequences do not seem to be well understood. When a task falls behind schedule for whatever reason, then depending upon the essentiality of the task, special efforts may be taken to make up lost time so as to meet its schedule as closely as possible. These special efforts inevitably cause a disruption in the shop's plans, which may take other tasks off-schedule which may lead to further expediting. Clearly a snowballing effect is possible. The need exists to increase our knowledge of how expediting can be done so as to achieve the best overall schedule performance.

Another specialized function is the releasing of tasks to the shop floor, especially for an open shop. Here for a given set of tasks, the desired objective may be to release the tasks to maximize facility utilization with the minimum amount of work-in-process inventory. Ideally, tasks are released so that high facility utilization is obtained, yet no tasks ever wait at any processing facility. Again there seems to be very little work dealing with this function.

F. Scheduling of Computerized Manufacturing Systems: Computerized manufacturing systems seem to be the wave of the future, especially in batch manufacturing. These systems are highly automated and consist of computer-controlled machining stations linked together by an automated material handling system. These systems are very complex and offer the potential for high productivity due to the systems' automation and flexibility. The scheduling of these systems would seem to be particular in that the scheduling system must operate in real time and need reflect both the manufacturing system's flexibility and inflexibility. System flexibility results from the generality and adaptability inherent in the machining stations; this flexibility creates many alternative routing possibilities for most tasks. The automated material handling system typically limits system flexibility as a result of the finite capacity of both the transport system between machining centers and the storage buffers at machining centers. Some preliminary work on scheduling these systems has been done by Stecke and Solberg [130]; this work needs to be extended and expanded so that the full potential of computerized manufacturing systems can be obtained.

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